National Defence / Défense nationale

SQUARE-LAW DEVICE ANALYSIS FOR THE DETECTION AND TIMING ESTIMATION OF A BURST BRSK SIGNAL (U)

by

W.R. Seed
SATCOM Section
Electronics Division

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
TECHNICAL NOTE 91-6

PCN
D6470

June 1991
Ottawa

92-02067/

ABSTRACT

There are a number of communications applications for which real-time processing of the input signal is a requirement. The initial detection of a binary phase-shift-keyed (BPSK) modulated frequency-hopped signal is one example. The digital signal processing workload is eased considerably if, instead of matched filtering the input waveform on a bit basis, the envelope of the waveform is matched filtered. Although this form of sub-optimal processing does ease the hardware requirements, there is a cost in terms of detection performance.

In this technical note, the use of a square-law device for the detection and timing estimation of a burst BPSK signal is analyzed. By evaluating the signal and noise power spectral densities at the devices's output, the baseband component is shown to provide a higher signal-to-noise ratio (*SNR*), and therefore more favorable detection/estimation statistics than the bandpass component. In addition, the *SNR* of a BPSK-modulated pulse following the envelope detector and subsequent lowpass filter is compared to that obtained from a CW pulse having equal power and duration. From this comparison, an expression is derived that transforms the E_b/N_o for a burst BPSK-modulated signal to an equivalent E_b/N_o for a CW pulse. This effective E_b/N_o can be utilized in parameter estimation and signal detection analyses for BPSK-modulated pulses.

RÉSUMÉ

Il existe plusieurs applications en communications pour lesquelles le traitement en temps réel du signal d'entrée est un besoin. La détection initiale d'un signal modulé en BPSK et avec sauts de fréquence en est un exemple. La quantité de traitement est considérablement réduite si, plutôt que filtrer adaptivement l'onde d'entrée et ainsi fournir un traitement pour un bit à la fois, c'est l'enveloppe de l'onde qui est filtrée adaptivement. Bien que cette forme non optimale de traitement facilite les besoins de matériel, le compromis est une baisse de la performance de détection.

Dans cette note technique, l'utilisation d'un dispositif de mise au carré pour la détection et l'estimation temporelle d'un signal en rafales de pulses binaires modulés en phase est analysée. En évaluant les densités spectrales de puissance du signal et du bruit à la sortie du dispositif, il est montré que la composante en bande de base fournie un meilleur rapport signal-bruit (*SNR*) et ainsi, de meilleures statistiques de détection et estimation que la composante en bande passante. De plus, le *SNR* d'un pulse binaire modulé en phase après le détecteur d'enveloppe et le subséquent filtre passe-bas est comparé à celui obtenu avec un pulse CW ayant la même puissance et la même durée. A partir de cette comparaison, une expression transformant le rapport E_b/N_o d'un signal BPSK en rafales à un rapport E_b/N_o equivalent pour un pulse CW est dérivée. Le rapport E_b/N_o effectif peut alors être utilisé pour les analyses d'estimation de paramètres et de détection de signal pour des pulses BPSK.

EXECUTIVE SUMMARY

A square-law device is often found in the receiver front-end of radar and communication systems. The output of this device is comprised of two components, one at baseband and the other at twice the input frequency. The former component is used typically to detect the presence of a signal (without requiring knowledge of the carrier phase). The latter component is commonly used to collapse the signal energy of a binary phase-shift keyed (BPSK) signal to a much narrower band for carrier phase estimation of the input signal. Although envelope detection of a burst BPSK-modulated signal is a form of sub-optimal processing, this technique can alleviate a considerable amount of the processing workload associated with a digital matched filter, and more easily facilitate real-time processing.

In this paper, the two outputs of a square-law device are analyzed in terms of output signal-to-noise ratio (*SNR*) for a burst BPSK-modulated pulse. After first deriving the device's output for a general BPSK signal, a closer examination is performed for a 13-bit Barker code which was selected for its good autocorrelation properties. The baseband component is shown to provide a 2-3 dB performance advantage over the bandpass component. This *SNR* advantage increases as the frequency uncertainty of the input signal and hence, the input filter bandwidth, increases. The *SNR* of a BPSK-modulated pulse following the envelope detector and lowpass filter is compared to that obtained from a CW pulse having an equal power and duration. From the comparison, an expression is derived that transforms the E_b/N_o value for a burst signal to an effective E_b/N_o value for a CW pulse. This effective E_b/N_o value can then be utilized in parameter estimation and signal detection analysis for BPSK-modulated pulses. The difference between the BPSK-modulated pulse E_b/N_o and effective CW pulse E_b/N_o illustrates the *SNR* cost of sub-optimal processing.

CONTENTS

ABSTRACT iii

RÉSUMÉ v

EXECUTIVE SUMMARY vii

1. INTRODUCTION 1

2. ANALYSIS 2
   2.1 DERIVATION OF SQUARE-LAW DEVICE OUTPUT 2
   2.2 SPECTRAL ANALYSIS OF THE OUTPUT SIGNAL 5

3. DISCUSSION 14

4. CONCLUSIONS 19

5. REFERENCES 20

1. INTRODUCTION

Square-law devices are routinely used in synchronization circuits. The output of the device has two components - one at baseband and another at twice the input signal carrier frequency. In practice, however, there are separate devices used to obtain the baseband or bandpass signal. An envelope detector is used when only the baseband portion is desired. The envelope detector is typically used for initial detection of the received signal, or for code tracking. When the primary output of the square-law device is the bandpass component, the device is called a frequency doubler. The frequency doubler is used in phase-locked loop applications to collapse the signal energy of a binary phase-shift-keyed (BPSK) signal to a much narrower band, as well as in frequency synthesizer applications.

There are a number of communications applications for which real-time processing of the input signal is a requirement. The initial detection of a BPSK-modulated frequency-hopped signal is one such application. The digital signal processing workload is eased considerably if, instead of matched filtering the input waveform on a bit basis, the envelope of the waveform is matched filtered. For an *N-bit* pulse, the savings in processing time is generally much greater than a factor of N. Although this form of sub-optimal processing does ease the hardware requirements, there is a penalty to be paid in terms of detection/estimation performance.

In this technical note, the use of a square-law device for the detection and timing estimation of a burst BPSK signal is analyzed. By evaluating the signal and noise power spectral densities at the devices's output, the baseband component is shown to provide a higher signal-to-noise ratio (*SNR*), and therefore more favorable detection/estimation statistics than the bandpass component. In addition, the *SNR* of a BPSK-modulated pulse at the output of the lowpass filter following the envelope detector is compared to that obtained from a CW pulse having an equal power and duration. From this comparison, an expression is derived that transforms the energy-per-bit-to-noise-power-spectral-density ratio, E_b/N_o, for a burst BPSK-modulated signal to an equivalent E_b/N_o for a CW pulse. This effective E_b/N_o can be utilized in parameter estimation and signal detection analyses for BPSK-modulated pulses. The difference between the BPSK-modulated pulse E_b/N_o and effective CW pulse E_b/N_o illustrates the *SNR* cost of sub-optimal processing. The analysis is aimed specifically at the detection of a burst BPSK signal, although it may be extrapolated to cover *M-ary* PSK.

In Section 2, the spectral density of the square-law device output is derived. The output *SNR* of both the baseband and bandpass components is determined as a function of the input filter bandwidth and E_b/N_o. A discussion of the results is provided in Section 3. The two square-law device output signals are compared in terms of output *SNR*. The lowpass component is then compared to a CW pulse having equal power and duration.

2. ANALYSIS

2.1 DERIVATION OF SQUARE-LAW DEVICE OUTPUT

Consider the receiver configuration shown in Fig. 1. The received signal plus bandpass noise, $x(t)$, is input to a square-law device. The output from this device, $y(t)$, is separated into two parallel branches for baseband and bandpass processing. The baseband component is lowpass filtered using $H_l(t)$ to produce $y_l(t)$ while the bandpass component is first filtered using $H_b(t)$, and then matched filtered to produce $z(t)$.

The signal $x(t)$ consists of a BPSK-modulated signal $s(t)$ having modulation $m(t)$ and power P, and bandpass noise $n(t)$,

$$x(t) = s(t) + n(t) \tag{1}$$

where

$$s(t) = \begin{cases} \sqrt{2P}m(t)\sin[2\pi f_o t + \Theta] & t_o \le t \le t_o + NT_b \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

and

$$n(t) = n_c(t)\cos(2\pi f_o t) - n_s(t)\sin(2\pi f_o t) \tag{3}$$

The modulation signal is non-zero for N consecutive bits, each of duration T_b. The input filter bandwidth W_o is assumed to be wide enough to cause minimal degradation to $m(t)$. f_o is the carrier frequency and Θ is an arbitrary initial phase. $n_c(t)$ and $n_s(t)$ are low frequency random processes, which are Gaussian, stationary, and statistically independent with a zero mean and two-sided power spectral density $N_o/2$. Furthermore, $n_c(t)$ and $n_s(t)$ are uniform in the $-\frac{1}{2}W_o$ to $+\frac{1}{2}W_o$ band, as shown in Fig. 2. The power in the two processes is the same and the power of each process taken individually is equal to the power of the process $n(t)$,

$$E[n_c^2(t)] = E[n_s^2(t)] = E[n^2(t)] = N_o W_o/2 \tag{4}$$

where $E[\cdot]$ is the expected value. Furthermore, since the noise processes have a zero mean, their variance is described by

$$\sigma_n^2 = \sigma_{nc}^2 = \sigma_{ns}^2 = N_o W_o/2 \tag{5}$$

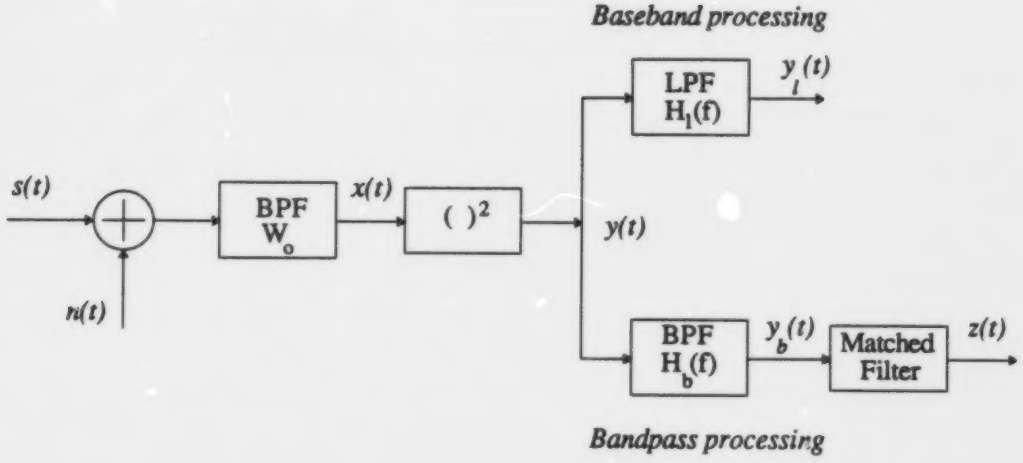

Fig. 1. Two parallel processing alternatives for the output of a square-law device.

A more convenient form of writing $n(t)$ when working with square-law devices is given by [1,p.159],

$$n(t) = N(t)\cos[2\pi f_o t + \phi(t)] \tag{6}$$

where

$$N(t) = [n_c^2(t) + n_s^2(t)]^{1/2} \tag{7}$$

and

$$\phi(t) = \tan^{-1}[n_s(t)/n_c(t)] \tag{8}$$

$N(t)$ is the envelope of the input noise process while $\phi(t)$ is its random phase.

Equation *(1)* can then be written as

$$x(t) = \sqrt{2P}m(t)\sin[2\pi f_o t + \Theta] + N(t)\cos[2\pi f_o t + \phi(t)] \tag{9}$$

Consequently, the output of the square-law device is given by

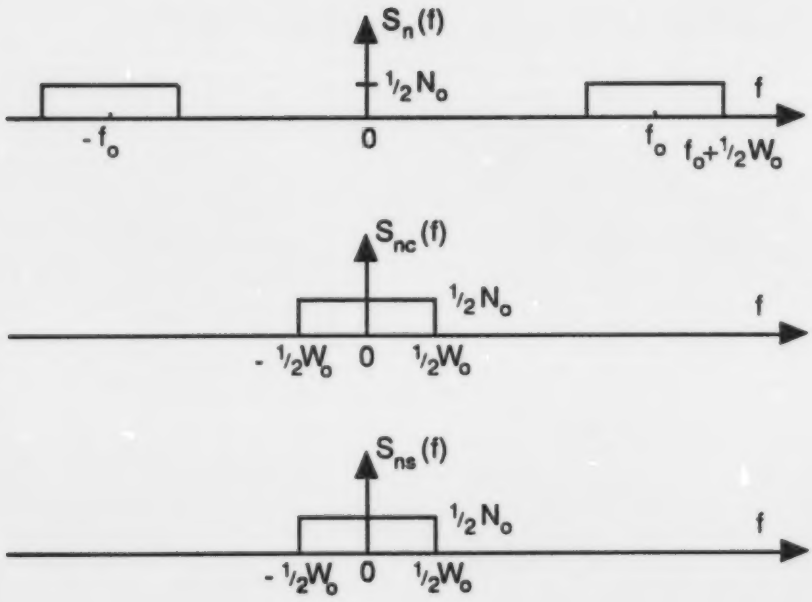

Fig. 2. Power spectral densities of noise processes $n(t)$, $n_c(t)$ and $n_s(t)$.

$$y(t) = 2P\sin^2[2\pi f_o t + \Theta] + N^2(t)\cos^2[2\pi f_o t + \phi(t)] + 2\sqrt{2P}m(t)N(t)\sin[2\pi f_o t + \Theta]\cos[2\pi f_o t + \phi(t)] \quad (10)$$

Since the input processes are independent, the mean of the output is

$$E[y(t)] = E[s^2(t)] + E[n^2(t)]$$

Since the input processes are also stationary, the mean becomes

$$E[y(t)] = P + \sigma_n^2 \quad (11)$$

2.2 SPECTRAL ANALYSIS OF THE OUTPUT SIGNAL

One method of comparing the performance of the upper and lower branches shown in Fig. 1 is to compare the spectral densities of the detected signal and noise. In order to derive the spectral densities, the autocorrelation function of the output signal is first computed, followed by the calculation of its Fourier transform.

The autocorrelation function of the output of the square-law device, $y(t)$, is

$$R_y(t_1,t_2) = E[(s(t_1)+n(t_1))^2(s(t_2)+n(t_2))^2]$$

Denoting $s(t_i)$ as s_i and $n(t_i)$ as n_i, the autocorrelation function for the output becomes

$$R_y(t_1,t_2) = E[s_1^2 s_2^2] + 4E[s_1 s_2]E[n_1 n_2] + E[s_1^2]E[n_2^2] + E[s_2^2]E[n_1^2] + E[n_1^2 n_2^2] \quad (12)$$

Defining $\tau=t_2-t_1$, and noting that the processes are stationary, *(12)* is written as

$$R_y(\tau) = R_{s^2}(\tau) + 4R_s(\tau)R_n(\tau) + 2R_s(0)\sigma_n^2 + R_{n^2}(\tau)$$

where

$$R_{s^2}(\tau) = E[s_1^2 s_2^2]$$

and

$$R_{n^2}(\tau) = E[n_1^2 n_2^2]$$

As with any nonlinear device, the autocorrelation function consists of three types of terms; signal×signal (*pronounced signal-cross-signal*), signal×noise and noise×noise,

$$R_y(\tau) = R_{s\times s}(\tau) + R_{s\times n}(\tau) + R_{n\times n}(\tau) \quad (13)$$

where

$$R_{s\times s}(\tau) = R_{s^2}(\tau) \quad (14)$$

is the signal×signal autocorrelation function,

$$R_{s\times n}(\tau) = 4R_s(\tau)R_n(\tau) + 2R_s(0)\sigma_n^2 \quad (15)$$

is the signal×noise autocorrelation function, and

$$R_{n\times n}(\tau) = R_{n^2}(\tau) \tag{16}$$

is the noise×noise autocorrelation function. $R_s(\tau)$ and $R_n(\tau)$ are the autocorrelation functions of the input signal and noise processes, respectively.

The spectral density of the output of the square-law device, $S_y(f)$, is obtained by taking the Fourier transform, $\mathcal{F}\{\cdot\}$, of $R_y(\tau)$,

$$S_y(f) = S_{s\times s}(f) + S_{s\times n}(f) + S_{n\times n}(f) \tag{17}$$

where

$$S_{s\times s}(f) = \mathcal{F}\{R_{s\times s}(\tau)\},$$

$$S_{s\times n}(f) = \mathcal{F}\{R_{s\times n}(\tau)\},$$

and

$$S_{n\times n}(f) = \mathcal{F}\{R_{n\times n}(\tau)\}$$

Signal×Signal Term

Substituting equation *(2)* in *(14)* yields

$$R_{s\times s}(\tau) = E[2PRect\{t/(NT_b)\}\sin^2(2\pi f_o t+\Theta)\,2PRect\{(t+\tau)/(NT_b)\}$$
$$\cdot(\sin^2(2\pi f_o(t+\tau)+\Theta))] \tag{18}$$

where

$$Rect\{t/(NT_b)\} = \begin{cases} 1 & 0 \le t \le NT_b \\ 0 & elsewhere \end{cases} \tag{19}$$

is the rectangular function which is unity over the duration of the *N-bit* pulse and zero elsewhere. Equation *(18)* is thus simplified as

$$R_{s\times s}(\tau) = P^2R_{Rect}\{\tau/(NT_b)\} + \tfrac{1}{2}P^2R_{Rect}\{\tau/(NT_b)\}\cos(4\pi f_o\tau) \tag{20}$$

with $R_{Rect}\{\tau/(NT_b)\}$ the autocorrelation function of $Rect\{t/(NT_b)\}$. $R_{Rect}\{\tau/(NT_b)\}$ is given by the triangular function, $\Lambda\{\tau/(NT_b)\}$,

$$\Lambda\{\tau/(NT_b)\} = \begin{cases} 1-|\tau|/(NT_b) & |\tau| \le NT_b \\ 0 & elsewhere \end{cases} \qquad (21)$$

An important observation to made from *(20)* is that the power of the first term is twice that of the second term. In a later section it will be shown that this factor of two results in the baseband component outperforming the bandpass component for all values of E_b/N_o.

Taking the Fourier transform of *(20)*,

$$\begin{aligned} S_{s\times s}(f) &= P^2NT_b\,Sinc^2\{\pi fNT_b\} + \tfrac{1}{4}P^2NT_b\,\delta(f+2f_o) * Sinc^2\{\pi fNT_b\} \\ &\quad + \tfrac{1}{4}P^2NT_b\,\delta(f-2f_o) * Sinc^2\{\pi fNT_b\} \\ &= P^2NT_b\,Sinc^2\{\pi fNT_b\} + \tfrac{1}{4}P^2NT_b\,Sinc^2\{\pi(f+2f_o)NT_b\} \\ &\quad + \tfrac{1}{4}P^2NT_b\,Sinc^2\{\pi(f-2f_o)NT_b\} \end{aligned} \qquad (22)$$

where $Sinc\{x\} = \sin(x)/x$, $\delta()$ is the unit impulse function and * denotes convolution.

Signal×Noise Term

To obtain an exact expression for the autocorrelation function of the signal×noise term, given in *(15)*, $R_s(\tau)$ must first be derived,

$$R_s(\tau) = 2PE[m(t)m(t+\tau)\cos(2\pi f_o t + \Theta)\cos(2\pi f_o(t+\tau) + \Theta)]$$

which simplifies to

$$R_s(\tau) = PR_m(\tau)\cos(2\pi f_o \tau) \qquad (23)$$

$R_m(\tau)$ is the autocorrelation function of the modulation signal.

For the present analysis, an *N-bit* Barker code is selected because of its good autocorrelation properties (i.e., low sidelobe levels). The autocorrelation function for the Barker code can be closely approximated by the triangular function, $\Lambda\{\tau/T_b\}$, where

$$\Lambda\{\tau/T_b\} = \begin{cases} 1-|\tau|/T_b & |\tau| \le T_b \\ 0 & elsewhere \end{cases} \qquad (24)$$

with T_b denoting the bit period. Strictly speaking, the autocorrelation function is not zero for values of τ greater than $\pm T_b$. The sidelobes oscillate between zero and $1/N$ with each T_b increase in τ. For $N=13$, the sidelobes are 22 dB below the main lobe; the triangular function

is therefore a reasonable approximation for the Barker code autocorrelation function. Equation *(23)* thus becomes

$$R_s(\tau) = P\Lambda\{\tau/T_b\}\cos(2\pi f_o\tau) \qquad (25)$$

which, when evaluated at $\tau=0$, yields

$$R_s(0) = P \qquad (26)$$

Substituting equations *(25)* and *(26)* into *(15)*,

$$R_{s\times n}(\tau) = 4P\Lambda\{\tau/T_b\}\cos(2\pi f_o\tau)R_n(\tau) + 2P\sigma_n^2 \qquad (27)$$

The spectral density of the signal×noise term, *(27)*, is

$$S_{s\times n}(f) = 4S_s(f)*S_n(f) + 2P\sigma_n^2\delta(f) \qquad (28)$$

The spectral density of the input signal is

$$\begin{aligned} S_s(f) &= \tfrac{1}{2}PT_b\,\mathbf{Sinc}^2\{\pi fT_b\}*[\delta(f-f_o)+\delta(f+f_o)] \\ &= \tfrac{1}{2}PT_b\,\mathbf{Sinc}^2\{\pi(f-f_o)T_b\} + \tfrac{1}{2}PT_b\,\mathbf{Sinc}^2\{\pi(f+f_o)T_b\} \end{aligned} \qquad (29)$$

The noise power spectral density is written as

$$\begin{aligned} S_n(f) &= \tfrac{1}{2}N_o\,\mathbf{Rect}\{|f|/(\tfrac{1}{2}W_o)\}*[\delta(f+f_o)+\delta(f-f_o)] \\ &= \tfrac{1}{2}N_o\,\mathbf{Rect}\{|f+f_o|/(\tfrac{1}{2}W_o)\} + \tfrac{1}{2}N_o\,\mathbf{Rect}\{|f-f_o|/(\tfrac{1}{2}W_o)\} \end{aligned} \qquad (30)$$

Substituting *(29)* and *(30)* in *(28)* yields

$$\begin{aligned} S_{s\times n}(f) = 2PT_b[&\mathbf{Sinc}^2\{\pi(f+f_o)T_b\} + 2P\sigma_n^2\delta(f) + \mathbf{Sinc}^2\{\pi(f-f_o)T_b\}] \\ &*[\tfrac{1}{2}N_o\,\mathbf{Rect}\{|f+f_o|/(\tfrac{1}{2}W_o)\}+\tfrac{1}{2}N_o\,\mathbf{Rect}\{|f-f_o|/(\tfrac{1}{2}W_o)\}] \end{aligned} \qquad (31)$$

Defining the convolution of the $\mathbf{Rect}\{\cdot\}$ function with the $\mathbf{Sinc}^2\{\cdot\}$ function as

$$\alpha\{f\} = T_b\mathbf{Rect}\{|f|/(\tfrac{1}{2}W_o)\}*\mathbf{Sinc}^2\{\pi fT_b\} \qquad (32)$$

equation *(31)* becomes

$$S_{s\times n}(f) = PN_o[\alpha\{f+2f_o\}+2\alpha\{f\}+\alpha\{f-2f_o\}] + 2P\sigma_n^2\delta\{f\} \qquad (33)$$

The maximum value of $\alpha\{f\}$ is denoted by $\alpha_o = \alpha\{0\}$. For $W_o \geq 1/T_b$, it is easily shown that $\alpha_o \sim 1$. The first three terms of *(33)* represent the variable portion of the signal×noise term, centered at $-2f_o$, *0* and $2f_o$, respectively. The last term is an impulse signal at *0 Hz* that represents the product of the mean signal and noise powers. $S_s(f)$, $S_{s\times s}(f)$ and $S_{s\times n}(f)$ are illustrated in Fig. 3.

Noise×Noise Term

The autocorrelation function of the noise×noise term is obtained from *(6)* and *(16)*,

$$R_{n\times n}(\tau) = E^2[n^2] + 2R_{n^2}(\tau)$$

or, combining with *(5)*,

$$R_{n\times n}(\tau) = \sigma_n^4 + 2R_{n^2}(\tau) \qquad (34)$$

The spectral density of the noise×noise term, illustrated in Fig. 3(d), is given by

$$S_{n\times n}(f) = \sigma_n^4\delta(f) + 2S_n(f) * S_n(f)$$

$$= \sigma_n^4\delta(f) + (N_o^2/2)[\Lambda\{(f+2f_o)/W_o\} + 2\Lambda\{f/W_o\} + \Lambda\{(f-2f_o)/W_o\}] \qquad (35)$$

$S_{n\times n}(f)$ is comprised of terms representing the mean output value of the squared noise (an impulse at *0*) and the variable portion of the squared noise centered at *0* and $\pm 2f_o$.

As shown in Fig. 1, the difference between the two branches is in the filtering of $y(t)$. Consider first the lowpass portion of $y(t)$, denoted as $y_l(t)$. A lowpass filter, $H_l(f)$, is used to match filter the baseband signal. (A notch filter at *dc* is typically used to remove the impulse noise components of $y_l(t)$). Fig. 4 illustrates the resulting spectral densities of the signal and noise terms of $y_l(t)$. The bandpass signal $y_b(t)$ is bandlimited by a filter, $H_b(f)$, which is the bandpass equivalent of $H_l(f)$. The spectral density of $y_b(t)$ is shown in Fig. 5.

In order to select the better of the two detection schemes, it is necessary to compare their performance. One quantity that may be used to compare the two candidates is *SNR*.

The bandwidth of the (front-end) bandpass filter is defined as $W_o = \xi/T_b$ where ξ is a factor on the order of unity which relates the filter bandwidth to the bit rate, $1/T_b$. Assuming that N is large, the spectral densities of the noise×noise and signal×noise terms of $y_l(t)$ within a bandwidth of $1/(NT_b)$ are approximately flat. Referring to Fig. 4, the *SNR* of $y_l(t)$ is readily obtained as

$$SNR(y_l) = \frac{P^2}{(N_o^2 W_o + 2PN_o)/(NT_b)} \qquad (36)$$

Noting that $P=E_b/T_b$ and that $W_o=\xi/T_b$, (36) is simplified as

$$SNR(y_l) = \frac{N(E_b/N_o)^2}{\xi + 2E_b/N_o} \qquad (37)$$

The aggregate noise power at $\pm 2f_o$ is equal to the variable portion at *dc*; however, as earlier derived in *(20)* the aggregate signal power at $\pm 2f_o$ is only one-half that of the *dc* portion. The *SNR* for the bandpass version of *y(t)* is therefore one-half of the lowpass *SNR*,

$$SNR(y_b) = \frac{N(E_b/N_o)^2}{2\xi + 4E_b/N_o} \qquad (38)$$

$SNR(y_l)$ is thus observed to exceed $SNR(y_b)$ for all values of E_b/N_o. The loss in *SNR* for the bandpass signal cannot be recovered; consequently, the baseband portion is the better choice for subsequent detection and estimation.

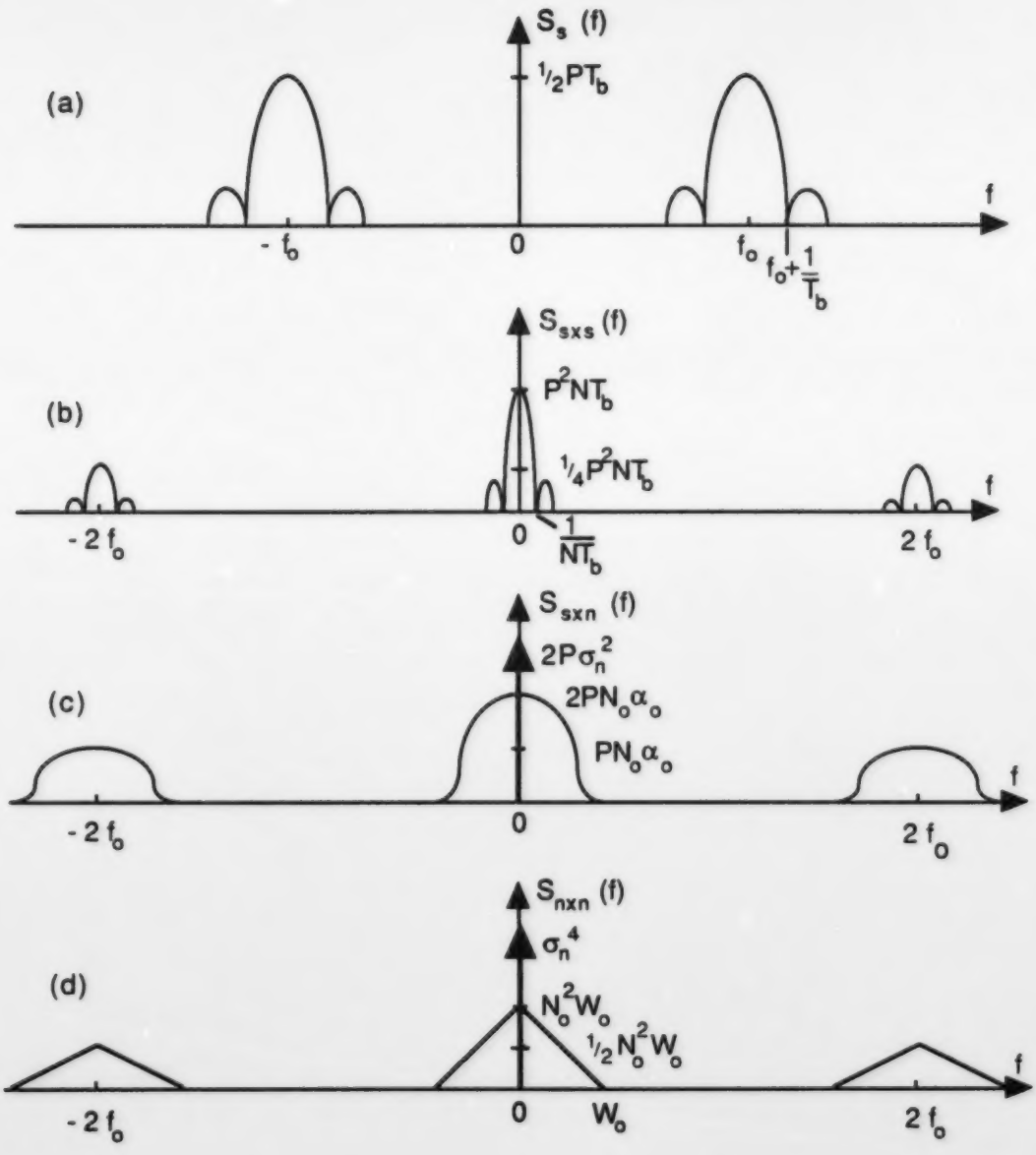

Fig. 3. Spectral densities of (a) input signal, (b) output signal×signal, (c) output signal×noise, and (d) output noise×noise terms.

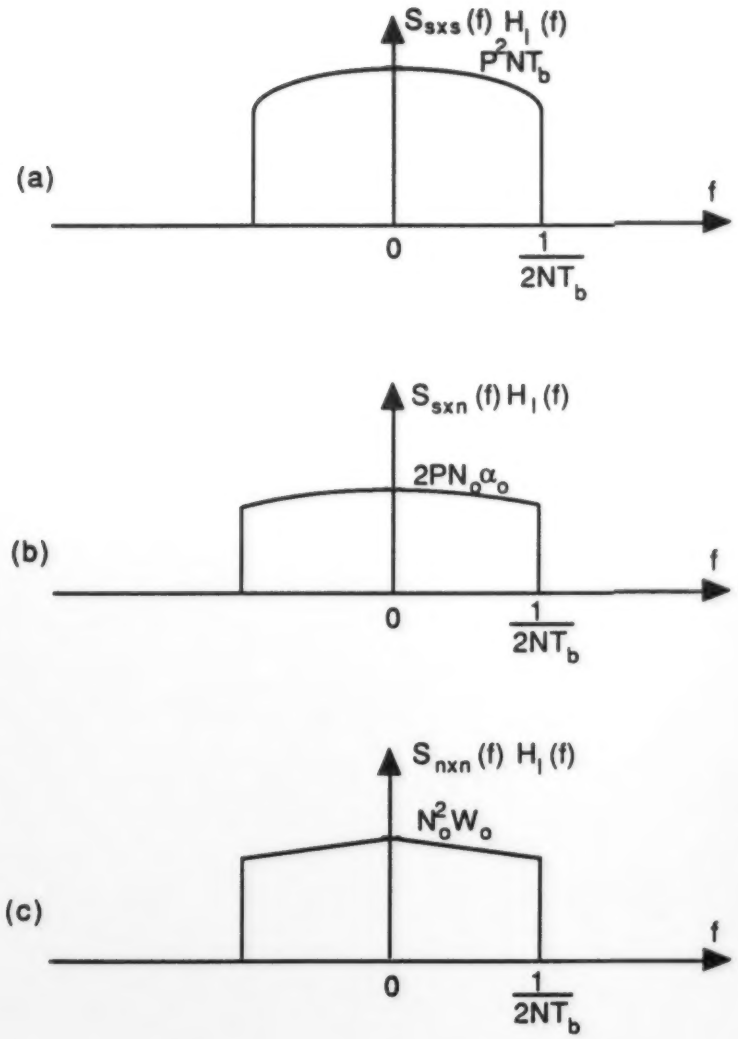

Fig. 4. Spectral density of variable $y_l(t)$ components: (a) signal × signal, (b) signal × noise, and (c) noise × noise terms.

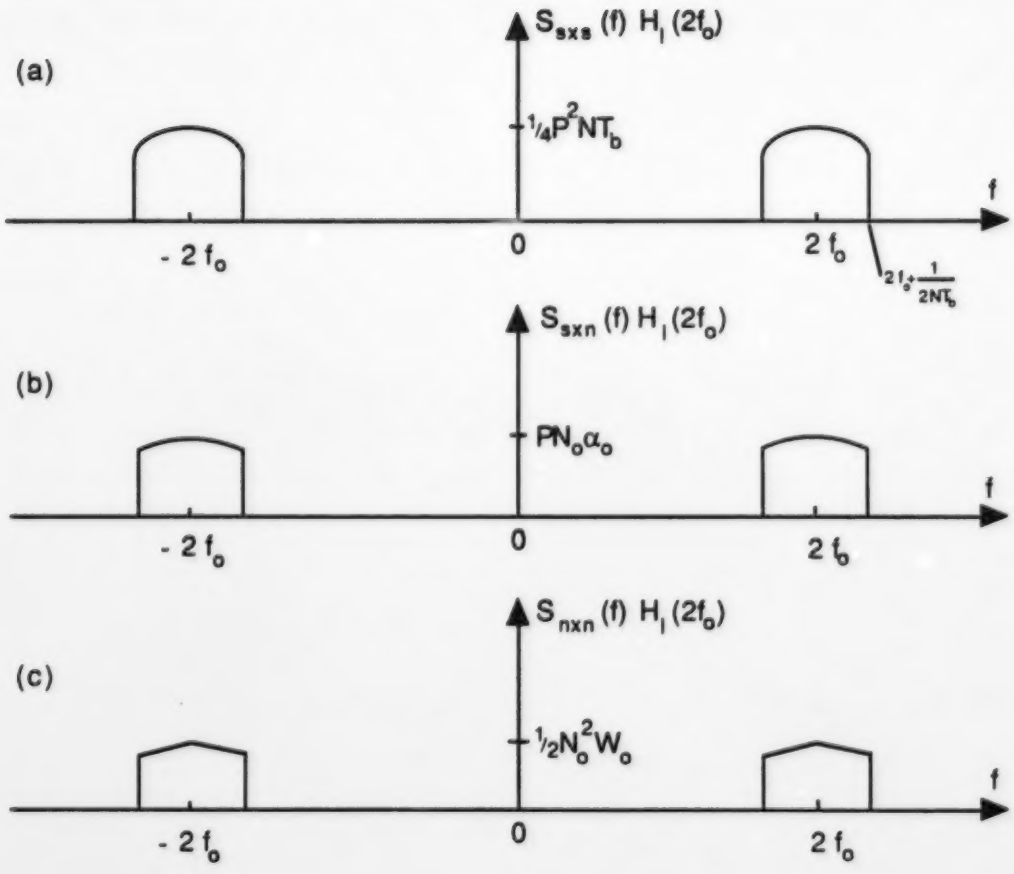

Fig. 5. Spectral density of variable $y_b(t)$ components: (a) signal × signal, (b) signal × noise, and (c) noise × noise terms.

3. DISCUSSION

A measure of the filter output *SNR*, while being a convenient, tractable quantity, may not clearly indicate the worth of a nonlinear detector. It is the process end statistic that is paramount - bit error rate (*BER*) when considering data demodulation, and probability of detection and false alarm rate when considering signal detection. For example, consider the output *SNR* following the detection of a CW pulse for two envelope detectors; a linear device and a square-law device, all other parameters being equal. It has been shown that for high input *SNR* the detection performance for each device is identical [2,p.384]. Upon further analysis, however, the square-law device is observed to provide an output *SNR* which is 6 dB below that of the linear envelope detector [3,1094]. The reason for the difference in *SNR* is because of the difference in statistics between the two detection schemes. The output *SNR* is thus seen to be a poor indicator of the performance of dissimilar systems.

A comparison of the *SNR* for bandpass and lowpass versions of $y(t)$ considered in the previous section is reasonable, nonetheless, since the only difference between them is a linear process (i.e., bandpass versus lowpass filtering), and the input and output *SNRs* are not being compared directly. Table I provides a comparison of the output *SNR* for $y_l(t)$ and $y_b(t)$ for a range of input E_b/N_o with $N=13$ and $\xi=1$ The output *SNR* of $y_b(t)$ is *3 dB* below that of $y_l(t)$. To achieve the same output *SNR*, the loss when referred to the input E_b/N_o, for $E_b/N_o>0$ *dB*, varies from *2.3* to *2.8 dB*, as shown in Fig. 6.

It is useful to compare the output *SNR* of a BPSK-modulated pulse to a CW pulse assuming each has a duration of NT_b, power E_b/T_b, lowpass filter bandwidth of $1/(NT_b)$ and $\xi=1$. The signal×signal term will be the same for both the modulated and CW pulse. The peak spectral density is given by P^2NT_b, the first nulls are at $\pm 1/(NT_b)$, and the power is P^2.

E_b/N_o (dB)	SNR(y_l) (dB)	SNR(y_b) (dB)
-6	-2.7	-5.7
-3	2.1	-0.9
0	6.4	3.4
3	10.2	7.2
6	13.6	10.6
10	17.9	14.9
13	21.0	18.0

Table I. Output SNR of square-law detector for baseband and bandpass components.

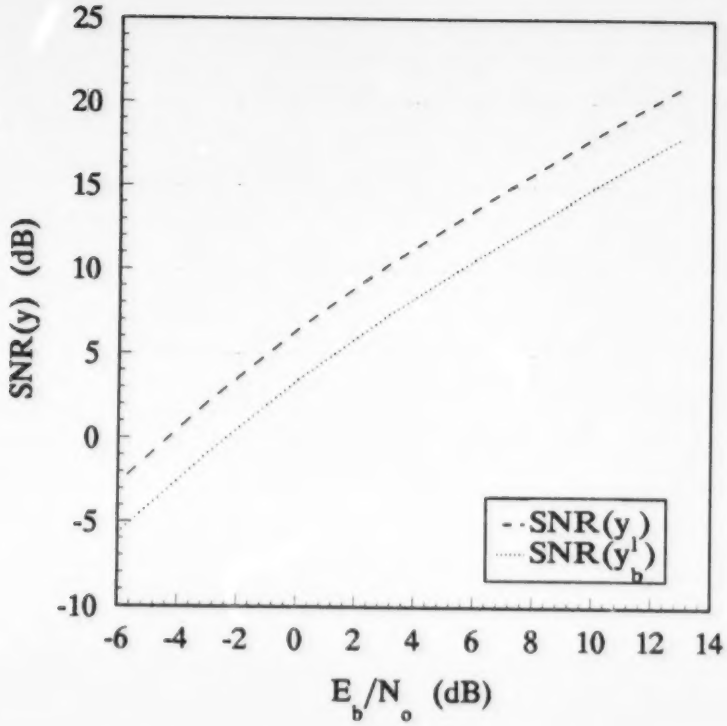

Fig. 6. Comparison of $SNR(y_l)$ and $SNR(y_{l_b})$ as a function of E_b/N_o.

The noise×noise term will be different for the two pulses. Since the noise×noise term, by definition, is the convolution of the input noise spectral density with itself, the peak spectral density is proportional to the input filter bandwidth. The modulated pulse requires an input filter bandwidth approximately N times that of the CW pulse. The bandwidth of the lowpass filter is the same for each pulse, $1/(NT_b)$; consequently, the power of the noise×noise term for the modulated pulse will be approximately N times that of the CW pulse. It should be noted that both the input filter bandwidth and the lowpass filter bandwidth influence the level of the noise×noise power.

The spectrum of the signal×noise term will also be different for the two pulses. The signal and bandpass noise spectrums for the modulated pulse are wider by a factor of N than the corresponding spectrums of the CW pulse. The signal spectral density for the modulated pulse is, however, only $1/N^{th}$ that of the CW pulse. The convolution of the signal and noise spectrums results in a peak spectral density that is identical for the two pulses. The primary difference between the signal and noise terms is that the spectral width of the modulated pulse

is approximately N times that of the CW pulse. Since both detected pulses pass through identical lowpass filters, the output signal×noise power will be nearly the same. An important observation to be made here is that the input bandpass filter bandwidth does not affect the signal×noise component of $y_l(t)$. This fact is significant in that it allows the input filter bandwidth to be wide enough to pass the signal modulation with minimal distortion yet does not degrade the output SNR under high E_b/N_o conditions, where the signal×noise term dominates the noise expression.

The above performance comparison between the modulated and CW pulses can also be illustrated by considering their output SNR at high and low E_b/N_o values. The SNR for the lowpass portion of the modulated pulse $y(t)$ was derived earlier as, assuming $\xi=1$,

$$SNR_{MOD}(y_l) = \frac{N(E_b/N_o)^2}{1+2(E_b/N_o)} \tag{37}$$

while for a CW pulse [1,p.266]

$$SNR_{CW}(y_l) = \frac{SNR_i^2}{1+2SNR_i} \tag{39}$$

where

$$SNR_i = N(E_b/N_o)$$

is the input SNR. Substituting for SNR_i,

$$SNR_{CW}(y_l) = \frac{N^2(E_b/N_o)^2}{1+2N(E_b/N_o)} \tag{40}$$

When $E_b/N_o \gg 1$, the output SNR for both pulses becomes

$$SNR_{MOD}(y_l) = SNR_{CW}(y_l) = \tfrac{1}{2}N(E_b/N_o) \tag{41}$$

Equation (41) shows that for high E_b/N_o, the phase modulation becomes inconsequential. This result is expected since the signal×noise power, which dominates the noise expression for high E_b/N_o, is equal for the two pulses.

When E_b/N_o is low

$$SNR_{MOD}(y_l) = N(E_b/N_o)^2 \tag{42}$$

while for the CW pulse,

$$SNR_{CW}(y_1) = N^2(E_b/N_o)^2 \tag{43}$$

Since the power of the noise×noise term dominates the noise expression in a low E_b/N_o environment, the output *SNR* of the CW pulse is expected to be N times that of the modulated pulse.

The output *SNR* for the modulated and CW pulses, assuming $\xi=1$ and $N=13$, calculated using *(37)* and *(40)*, respectively, are listed as a function of E_b/N_o in Table II and illustrated in Fig. 7. The output *SNR* of the BPSK-modulated pulse approaches that of the CW pulse when $E_b/N_o \gg 0$ *dB*.

An expression can be derived that transforms the E_b/N_o for a BPSK-modulated pulse to the effective E_b/N_o of a CW pulse. This is accomplished by equating *(37)* and *(40)*,

$$\frac{N\rho_{MOD}}{1+2\rho_{MOD}} = \frac{N^2\rho_{CW}}{1+2N\rho_{CW}} \tag{44}$$

where $\rho_{MOD}=(E_b/N_o)$ for a modulated pulse and $\rho_{CW}=(E_b/N_o)$ for a CW pulse. Solving for ρ_{CW} and performing some algebraic manipulation, one obtains,

$$\rho_{CW} = \frac{\rho_{MOD}^2+\rho_{MOD}^2\sqrt{1+1/(N\rho_{MOD}^2)+2/(N\rho_{MOD})}}{1+2\rho_{MOD}} \tag{45}$$

For example, given a modulated pulse with $N=13$ and $\rho_{MOD}=10$ *dB*, the effective E_b/N_o for a CW pulse is $\rho_{CW}=9.81$ *dB*.

E_b/N_o (dB)	$SNR_{MOD}(y_1)$ (dB)	$SNR_{CW}(y_1)$ (dB)
-6	-2.7	1.5
-3	2.1	4.8
0	6.4	8.0
3	10.2	11.1
6	13.6	14.1
10	17.9	18.1

Table II. Output SNR of BPSK-modulated and CW pulses.

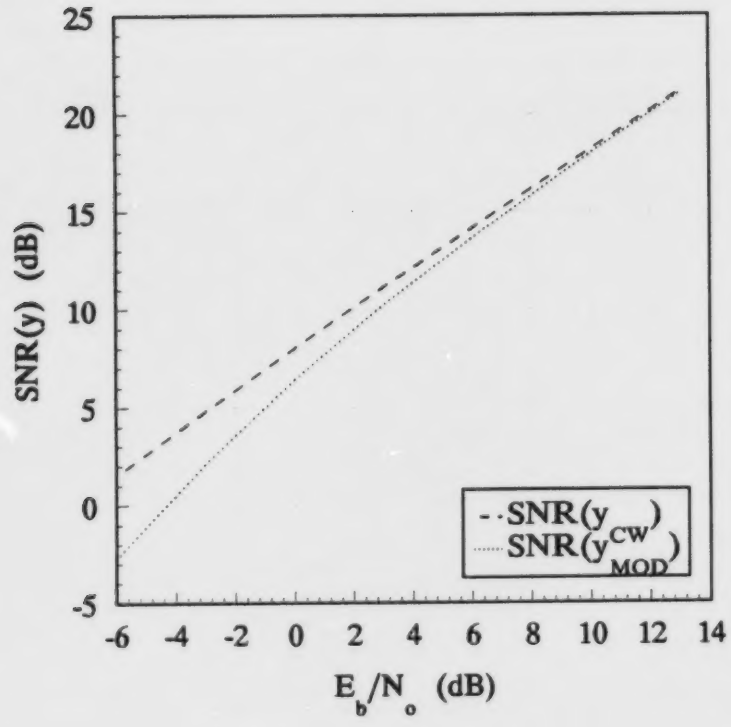

Fig. 7. Comparison of output *SNR* versus E_b/N_o for CW and BPSK-modulated pulses.

4. CONCLUSIONS

The preceeding analysis shows that it is the lowpass portion of the square-law device that should be used for the detection of a BPSK signal. Furthermore, when $E_b/N_o \gg 1$ the square-law device output *SNR* for a BPSK-modulated pulse is nearly equal to that of a CW pulse having equal energy and duration. An expression has been derived that transforms the E_b/N_o for a BPSK-modulated pulse to the effective E_b/N_o of a CW pulse for detection and estimation analyses.

5. REFERENCES

1. W.B. Davenport and W.L. Root, AN INTRODUCTION TO THE THEORY OF RANDOM SIGNALS AND NOISE, IEEE Press, New York, 1987.

2. M.I. Skolnik, INTRODUCTION TO RADAR SYSTEMS, McGraw-Hill Book Company, New York, 1980.

3. G.L. Turin, "An Introduction to Digital Matched Filters", Proceedings of the IEEE, Vol. 64, No. 7, July 1976.

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. **ORIGINATOR** (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.)

Defence Research Establishment Ottawa
Ottawa, Ontario
K1A 0Z4

2. **SECURITY CLASSIFICATION** (overall security classification of the document including special warning terms if applicable)

UNCLASSIFIED

3. **TITLE** (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C or U) in parentheses after the title.)

Square-Law Device Analysis For The Detection And Timing Estimation Of A Burst BPSK Signal (U) (U)

4. **AUTHORS** (Last name, first name, middle initial)

W. R. Seed

5. **DATE OF PUBLICATION** (month and year of publication of document)

May 91

6a. **NO. OF PAGES** (total containing information. Include Annexes, Appendices, etc.)

25

6b. **NO. OF REFS** (total cited in document)

3

7. **DESCRIPTIVE NOTES** (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.

DREO Technical Note

8. **SPONSORING ACTIVITY** (the name of the department project office or laboratory sponsoring the research and development. Include the address.

EHF SATCOM Project Office, Electronics Division, Defence Research Establishement Ottawa
Ottawa, Ontario, K1A 0Z4

9a. **PROJECT OR GRANT NO.** (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant)

Project D6470

9b. **CONTRACT NO.** (if appropriate, the applicable number under which the document was written)

10a. **ORIGINATOR'S DOCUMENT NUMBER** (the official document number by which the document is identified by the originating activity. This number must be unique to this document.)

DREO Technical Note 91-6

10b. **OTHER DOCUMENT NOS.** (Any other numbers which may be assigned this document either by the originator or by the sponsor)

11. **DOCUMENT AVAILABILITY** (any limitations on further dissemination of the document, other than those imposed by security classification)

(X) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; further distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other (please specify):

12. **DOCUMENT ANNOUNCEMENT** (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). however, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)

Unlimited Announcement

UNCLASSIFIED

SECURITY CLASSIFICATION OF FORM

13. **ABSTRACT** (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

(U) There are a number of communications applications for which real-time processing of the input signal is a requirement. The initial detection of a BPSK-modulated frequency-hopped signal is one such example. The processing workload is eased considerably if, instead of match filtering the input waveform and thereby provide processing on a bit basis, the envelope of the waveform is match filtered. Although this form of sub-optimal processing does ease the hardware requirements, there is a cost in terms of detection performance.

In this technical note, the use of a square-law device for the detection and timing estimation of a burst binary phase-shift-keyed (BPSK) signal is analyzed. By evaluating the signal and noise power spectral densities at the devices's output, the baseband component is shown to provide a higher *SNR* and therefore, more favorable detection/estimation statistics than the bandpass component. In addition, the *SNR* of a BPSK-modulated pulse following the envelope detector and subsequent lowpass filter is compared to that obtained from a CW pulse having an equal power and duration. From the comparison, an expression is derived that transforms the E_b/N_o for a burst BPSK-modulated signal to an equivalent E_b/N_o for a CW pulse. This effective E_b/N_o can then be utilized in parameter estimation and signal detection analyses for BPSK-modulated pulses.

14. **KEYWORDS, DESCRIPTORS or IDENTIFIERS** (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Square-law
Nonlinear
Estimation
Detection

UNCLASSIFIED

SECURITY CLASSIFICATION OF FORM